Краткие сообщения ОИЯИ №3[49]-91 *JINR Rapid Communications No.3[49]-91*
УДК 539.12.01

DESCRIPTION OF $\omega \to 3\pi$, $\phi \to 3\pi$, $\eta \to \pi^+\pi^-\gamma$ AND $K_L \to \pi^+\pi^-\gamma$ DECAYS WITHIN THE NAMBU — JONA — LASINIO MODEL

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The decays $\omega \to 3\pi$, $\phi \to 3\pi$, $\eta \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-\gamma$ via the quark loops of anomalous type are described within the Nambu – Jona – Lasinio model. The importance of nondiagonal $\pi - a_1$ transitions at legs of box quark diagrams and role of form factors of intermediate ρ-mesons in pole-type diagrams are shown.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Описание распадов $\omega \to 3\pi$, $\phi \to 3\pi$, $\eta \to \pi^+\pi^-\gamma$ и $K_L \to \pi^+\pi^-\gamma$ в модели Намбу — Иона — Ласинио

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В кварковой модели Намбу – Иона – Ласинио описаны распады $\omega \to 3\pi$, $\phi \to 3\pi$, $\eta \to \pi^+\pi^-\gamma$, $K_L \to \pi^+\pi^-\gamma$, идущие через кварковые петли аномального типа. Показана важная роль, которую играют недиагональные переходы вида πa_1 на внешних концах в четырехугольных кварковых диаграммах и формфакторы промежуточных ρ-мезонов в диаграммах полюсного типа.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

1. Introduction

The processes $\omega \to 3\pi$, $\phi \to 3\pi$, $\eta \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-\gamma$ have long been studied from different points of view both in theoretical and experimental particle physics. Recently, however, noticeable changes have appeared in the experimental data on $\omega \to 3\pi$ and $\eta \to \pi^+\pi^-\gamma$ decays (the width of the former decreased by 20% and that of the latter increased by 40%) [1]. Taking into account the new data, we intend to give a consistent description of the above processes within our quark model of the superconductivity type (QMST) [2]. This is a version of the well-known Nambu–Jona-Lasinio quark model [3]. All the main noticeable effects on the final results will be taken into account. Briefly, they are as follows:

1. Each process is described by anomalous quark diagrams of two types: an anomalous box quark diagram and a pole diagram with the intermediate ρ-meson that links the anomalous triangular diagram with the $\rho\pi\pi$ vertex (Fig.1).

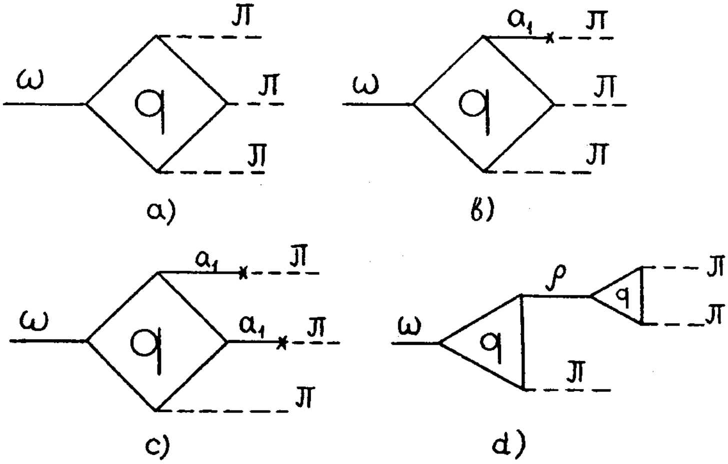

Fig.1

2. In the box diagrams nondiagonal π–a_1 transitions of (pion–axial-vector meson) on legs will be taken into account (Fig.1b,c). It greatly (four times) reduces their contribution to the amplitude.

3. In the pole diagrams form factors of ρ-mesons should be taken into account. It is especially important for the $\eta \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-\gamma$ processes, where the ρ-meson is far from its mass shell.

4. The ρ-meson width is of importance only in the $\omega \to 3\pi$ and $\phi \to 3\pi$ decays. Calculating the $K_L \to \pi^+\pi^-\gamma$ decay, we shall use the matrix elements of the $< K_L|\mathcal{L}^{eff}_W|\pi, \eta, \eta' >$ transitions obtained in our previous papers [4, 5].

2. The $\omega \to 3\pi$ and $\phi \to 3\pi$ Decays

In our model the $\omega \to 3\pi$ decay amplitude is determined by a set of contact diagrams with anomalous quark boxes (Fig.1a) and by pole diagrams with intermediate ρ-mesons (Fig.1d) linking two triangular quark loops, one of which is also anomalous ($\rho\omega\pi$). Thus, two of three vertices involved in these diagrams are described by the effective Wess–Zumino Lagrangian [6] [1]:

$$\mathcal{L}^{\Box}_{\omega \to 3\pi} = -\frac{iN_c g_\rho}{24\pi^2 f_\pi^3}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abc}\omega_\mu\partial_\nu\pi^a\partial_\alpha\pi^b\partial_\beta\pi^c, \tag{1}$$

$$\mathcal{L}^{\triangle}_{\omega\rho\pi} = -\frac{N_c g_\rho^2}{32\pi^2 f_\pi}\varepsilon^{\mu\nu\alpha\beta}\omega_{\mu\nu}\rho^a_{\alpha\beta}\pi^a, \tag{2}$$

[1] The Wess–Zumino Lagrangian is automatically obtained from the QMST in consideration of anomalous quark loops (see [7]).

where g_ρ is the $\rho \to \pi\pi$ decay constant ($\alpha_\rho = g_\rho^2/4\pi \cong 3$), $N_c = 3$ is the number of colours, $f_\pi = 93\ MeV$ is the pion decay constant, and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. The $\rho\pi\pi$ vertex is described by the Lagrangian

$$\mathcal{L}_{\rho\pi\pi} = g_\rho \varepsilon_{abc} \rho_\mu^a \pi^b \partial^\mu \pi^c. \quad (3)$$

Then, for the $\omega \to 3\pi$ decay amplitude we obtain

$$T_{\omega \to 3\pi} = \varepsilon_{\mu\nu\alpha\beta} \epsilon_\omega^\mu p_0^\nu p_-^\alpha p_+^\beta A(\omega \to 3\pi), \quad (4)$$

where

$$A(\omega \to 3\pi) = -\frac{N_c g_\rho}{4\pi^2 f_\pi^3} \left[1 + g_\rho^2 f_\pi^2 \sum_{i=+,0,-} \frac{1}{m_\rho^2 - (p - p_i)^2 + i m_\rho \Gamma_\rho} \right]. \quad (5)$$

Here the ρ-meson decay width $\Gamma_\rho = 150\ MeV$ is introduced; p_0, p_-, p_+ are the pion momenta, p is the ω-meson momentum.

For the $\omega \to 3\pi$ decay width we obtain the expression

$$\Gamma(\omega \to 3\pi) = \frac{m_\omega}{192\pi^3} \int \left[(\vec{p}_+)^2 (\vec{p}_-)^2 - (\vec{p}_+ \vec{p}_-)^2 \right] |A(\omega \to 3\pi)|^2 dE_+ dE_-. \quad (6)$$

The new experimental value of the $\omega \to 3\pi$ decay width is [1]

$$\Gamma^{exp}_{\omega \to 3\pi} = (7.49 \pm 0.09 \pm 0.05)\ MeV. \quad (7)$$

The theoretical value calculated with amplitude (5) is much larger than (7): $\Gamma^{theor}_{\omega \to 3\pi} = 15\ MeV$. This is because two more important effects are not taken into account in (5).

First of all, these are π–a_1 transitions[2] at the legs of the box diagrams [2] (Fig.1b,c). After removing the nondiagonal terms at the legs of the box diagrams we get an additional factor in the contribution from the contact term [2, 8]:

$$\Delta = 1 - 3(1 - Z^{-1}) + \frac{3}{2}(1 - Z^{-1})^2 \cong 0.25, \quad (8)$$

where $Z^{-1} = \frac{1}{2}[1 + \sqrt{1 - (2 g_\rho f_\pi / m_{a_1})^2}]$, $m_{a_1} = 1260\ MeV$ is the mass of the axial-vector meson a_1 [1]. This factor makes the contribution of contact diagrams four times smaller. As a result, we obtain the value 9.7 MeV for the width $\Gamma(\omega \to 3\pi)$, which is closer to the experimental one.

Allowance for π–a_1 transitions in pole diagrams (Fig.1d) does not result in any changes since they do not affect the anomalous $\omega\rho\pi$ vertices (see [10]), while the $\rho \to \pi\pi$ vertex (Lagrangian (3)) already includes them (see [2]).

[2]Like previous phenomenological chiral Lagrangians [9], the QMST contains nondiagonal terms of the form $c\vec{a}_1^\mu \partial_\mu \vec{\pi}$. To remove them, one should go over to physical axial-vector fields $\vec{a}_1^\mu = (\vec{a}_1^\mu)^{phys} - \frac{1 - Z^{-1}}{g_\rho f_\pi} \partial_\mu \vec{\pi}$. Then additional diagrams shown in Fig.1b,c appear in the $\omega \to 3\pi$ decay.

The last effect to be allowed for in derivation of the $A(\omega \to 3\pi)$ amplitude is the ρ-meson form factor, which appears when the $\rho \to \pi\pi$ vertex deviates from the ρ-meson mass shell. In the $\omega \to 3\pi$ decay it is of little importance, since the ρ-meson is close to its mass shell here. Yet, it is important in description of the $\eta \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-\gamma$ decays.

In ref. [2, 11, 12, 13] a ρ-meson form factor was proposed in the following form on the pion mass shell:

$$F_{\rho\pi\pi}(q^2) = \left(1 + \frac{q^2 - m_\rho^2}{8\pi^2 f_\pi^2}\right) \cong \frac{q^2}{m_\rho^2}, \qquad (8\pi^2 f_\pi^2 \cong m_\rho^2) . \tag{9}$$

This form factor is equal to one on the ρ-meson mass shell and tends to zero as q^2 at $q^2 \to 0$. The coefficient of q^2 is obtained by expanding the quark loop in terms of q^2 at small q^2. This behaviour of the form factor at small q^2 allows a correct estimation of the pion electromagnetic radius [13]. It also follows from the nonlocal version of the Nambu–Jona-Lasinio model [14]. Since the behaviour of the form factor in the region of q^2 close to m_ρ^2 is less studied, the weak effect of the form factor on the $\omega \to 3\pi$ decay allows a conclusion that in this energy region the form factor is close to one. Then, to match the behaviour of $F_{\rho\pi\pi}$ at small and large energies, one can introduce the switch function $\theta(a - q^2)$ in (9). For the optimal description of $\omega \to 3\pi$ and $\eta \to \pi^+\pi^-\gamma$ processes one should choose $a = (0.6m_\rho)^2$. Then form factor (9) takes the form

$$F_{\rho\pi\pi}(q^2) \cong 1 + \left(\frac{q^2}{m_\rho^2} - 1\right) \theta\left[(0.6m_\rho)^2 - q^2\right] . \tag{10}$$

Function (10) can be replaced by a smoother function without any significant changes in the results.

Now the amplitude $A(\omega \to 3\pi)$ looks like

$$A(\omega \to 3\pi) = -\frac{3g_\rho}{4\pi^2 f_\pi^3} \left\{ \Delta + g_\rho^2 f_\pi^2 \sum_{i=+,0,-} \frac{F_{\rho\pi\pi}[(p-p_i)^2]}{m_\rho^2 - (p-p_i)^2 + i m_\rho \Gamma_\rho} \right\} . \tag{11}$$

As a result, we obtain the decay width value that agrees with the experiment: $\Gamma(\omega \to 3\pi) = 7.1\ MeV$.

As is well known the $\phi \to 3\pi$ decay takes place because of ω-ϕ mixing: $\omega = \omega^{phys}\cos\alpha + \phi^{phys}\sin\alpha$, where $\alpha \approx 3^\circ$ is the ω-ϕ mixing angle. After the numerical integration in (6) (where we introduce $\sin\alpha$ and replace m_ω by m_ϕ) we obtain the following result for $\Gamma(\phi \to 3\pi)$:

$$\begin{aligned} \Gamma(\phi \to 3\pi) &= \Gamma^{cont} + \Gamma^{int} + \Gamma^{\rho-pol} \\ &= (0.0026 + 0.051 + 0.60 \cong 0.65)\ MeV. \end{aligned} \tag{12}$$

Here Γ^{cont}, Γ^{int} and $\Gamma^{\rho-pol}$ are the contributions of the contact term, interference term and ρ-pole term respectively. The influence of the form factor $F_{\rho\pi\pi}$

on $\Gamma(\phi \rightarrow 3\pi)$ is small here. We have $\Gamma(\phi \rightarrow 3\pi) = 0.68\ MeV$ when $F_{\rho\pi\pi} \equiv 1$. The experimental value [1]

$$\begin{aligned} \Gamma^{exp}_{tot}(\phi \rightarrow 3\pi) &= \Gamma(\phi \rightarrow \pi\rho) + \Gamma(\phi \rightarrow 3\pi) \\ &= (0.57 \pm 0.03)\ MeV + (0.084 \pm 0.053)\ MeV \qquad (13) \end{aligned}$$

is obtained from the incoherent addition of the widths into $\pi\rho$ and 3π. We see that the last term of this formula is mainly due to the interference term.

3. The $\eta \rightarrow \pi^+\pi^-\gamma$ Decay

The $\eta \rightarrow \pi^+\pi^-\gamma$ decay amplitude will be described by diagrams similar to those shown in Fig.1, but instead of the ω-meson there is the ρ^0-meson with its transition to the photon, and the neutron pion is replaced by the η-meson. These manipulations yield the following expression for the $\eta \rightarrow \pi^+\pi^-\gamma$ decay amplitude:

$$T_{\eta \rightarrow \pi^+\pi^-\gamma} = \varepsilon_{\mu\nu\alpha\beta}\epsilon^{\mu}_{\gamma} p^{\alpha}_{-} p^{\beta}_{+} p^{\nu}_{0} A(\eta \rightarrow \pi^+\pi^-\gamma), \qquad (14)$$

where

$$A(\eta \rightarrow \pi^+\pi^-\gamma) = \frac{e \sin\theta'}{4\pi^2 f^3_\pi} \left\{ \Delta + \frac{3g^2_\rho f^2_\pi F_{\rho\pi\pi}[(p_- + p_+)^2]}{m^2_\rho - (p_- + p_+)^2 + i m_\rho \Gamma_\rho} \right\}. \qquad (15)$$

Here e is the electric charge, $\theta' = \theta_0 - \theta$, θ_0 is the ideal mixing angle ($\theta_0 = 35.3°$), and $\theta = -18°$. As a result, the $\eta \rightarrow \pi^+\pi^-\gamma$ decay width is obtained to be

$$\begin{aligned} \Gamma(\eta \rightarrow \pi^+\pi^-\gamma) &= \frac{m^5_\pi m^2_\eta}{3(4\pi)^3} \int_0^{\frac{m^2_\eta - 4m^2_\pi}{2m_\eta m_\pi}} dx\, x^3 \left(\frac{m^2_\eta - 4m^2_\pi}{2m_\eta m_\pi} - x \right)^{\frac{3}{2}} \\ & \left(\frac{m_\eta}{2m_\pi} - x \right)^{-\frac{1}{2}} |A(\eta \rightarrow \pi^+\pi^-\gamma)|^2 = 58.5\ eV \qquad (16) \end{aligned}$$

The experimental value is [1] $\Gamma^{exp}_{\eta \rightarrow \pi^+\pi^-\gamma} = (58.07 \pm 5.86 \pm 1.78)\ eV$. Agreement is very good because, having in mind mainly this process where the form factor $F_{\rho\pi\pi}$ is of importance, we chose the switch parameter $a = (0.6 m_\rho)^2$ (see (10)).

4. The $K_L \rightarrow \pi^+\pi^-\gamma$ Decay

The $K_L \rightarrow \pi^+\pi^-\gamma$ decay is described by the pole diagrams shown in Fig.2. At first there is a transition of K_L to π^0, η, η' via weak interaction described by the effective Lagrangian $\mathcal{L}^{eff}_W$. Then these mesons decay into $\pi^+\pi^-\gamma$ through the above mode.

The effective Lagrangian $\mathcal{L}_W^{eff}$ has the form [4, 5, 15, 16]

$$\mathcal{L}_W^{eff} = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 Q_{\Delta s=1}, \tag{17}$$

where $Q_{\Delta s=1} = Q = Q_1 - 1.6Q_2 + 0.033Q_3 - 0.018Q_5 + 0.1Q_6$, $\frac{G_F}{\sqrt{2}} s_1 c_1 c_3 = 1.77 \cdot 10^{-6}\ GeV^{-2}$, $s_i = \sin\phi_i$, $c_i = \cos\phi_i$ are the elements of the Kobayashi-Maskawa matrix [17] and Q_i are the four-quark operators. As an example, we show here the most important "penguin"-type operator Q_6

$$Q_6 = [\bar{s}_a \gamma^\nu (1-\gamma_5) d_b] \sum_{q=u,d,s} [\bar{q}_b \gamma_\nu (1+\gamma_5) q_a].$$

Here $a, b = 1, 2, 3$ are the colour indices. Other operators can be found in ref. [4, 16]. Lagrangian (17) satisfies the selection rules $|\Delta S| = 1$, $|\Delta I| = 1/2, 3/2$.

Matrix elements of the $< K_L|Q|\pi^0, \eta, \eta' >$ transition are calculated in the two-loop approximation (see Fig.2) [4]. The diverging quark loops are in the low energy region, as in the QMST, and are cut off on the boundary of the region where spontaneous breaking of chiral symmetry occurs ($\Lambda = 1.25\ GeV$).

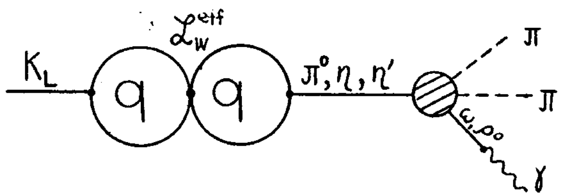

Fig.2

The constituent quark masses and the cut-off parameter correspond to the QMST [2] ($m_u = m_d = 280\ MeV$, $m_s = 460\ MeV$). As a result, we obtain the following values for the matrix elements of the transitions $K^0 \to \pi^0, \eta, \eta'$ at the angle $\theta = -18°$ (see details in ref. [4, 5]):

$$< \pi^0|Q|K^0 >= 4.9X, \qquad < \eta|Q|K^0 >= 3X, \qquad < \eta'|Q|K^0 >= -10.6X,$$

where $X =< \pi^0|Q_1|K^0 >= 3.5 \cdot 10^{-3}\ GeV^4$.

Then for the $K_L \to \pi^+\pi^-\gamma$ decay amplitude we obtain the expression

$$A(K_L \to \pi^+\pi^-\gamma) = \frac{ec}{4\pi^2 f_\pi^3} \left\{ \Delta + \frac{3g_\rho^2 f_\pi^2 F_{\rho\pi\pi}[(p_- + p_+)^2]}{m_\rho^2 - (p_- + p_+)^2 + i m_\rho \Gamma_\rho} \right\}, \tag{18}$$

where

$$c = G_F s_1 c_1 c_3 \left\{ \frac{< K_L|Q|\pi^0 >}{m_K^2 - m_\pi^2} + \sin\theta' \frac{< K_L|Q|\eta >}{m_K^2 - m_\eta^2} + \cos\theta' \frac{< K_L|Q|\eta' >}{m_K^2 - m_{\eta'}^2} \right\}.$$

The $K_L \to \pi^+\pi^-\gamma$ decay width is described by formula (16), where m_η is replaced by m_K. Thus, at $\theta = -18^\circ$ we obtain $\Gamma(K_L \to \pi^+\pi^-\gamma) = 4 \cdot 10^{-13}\ eV$, while the experimental value is $\Gamma^{exp}_{K_L \to \pi^+\pi^-\gamma} = (5.6 \pm 0.41 \pm 0.04) \cdot 10^{-13}\ eV$ [1]. Agreement within 30% is obtained. This agreement can be considered quite satisfactory because it is difficult to calculate the coefficient c with a good precision. Indeed, the matrix elements $< K_L|Q|\pi^0, \eta, \eta' >$ are calculated in a definite approximation with allowance for the mass difference of constituent quarks (u, d) and s, as pointed out above. Besides, the contributions from π, η, and η' mesons to c much compensate for each other:

$$c = 0.35 \cdot 10^{-7}[-\overset{(\pi)}{5.3} + \overset{(\eta)}{11.2} - \overset{(\eta')}{2.3} = 3.6] = 1.3 \cdot 10^{-7},$$

which considerably lowers the accuracy of calculations[3].

5. Conclusion

A unified description of three similar processes with allowance for the most important effects upon decay widths is attempted in the paper. Similarity of the $\omega \to 3\pi$, $\phi \to 3\pi$, $\eta \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-\gamma$ processes is due to fact that they all run through the anomalous quark loops (Fig.1). The investigations showed that allowance for π–a_1 transitions in the contact diagrams noticeably affects the final results, strongly reducing their contributions to the amplitudes. The ρ-meson width does not produce a noticeable effect on the decay width, especially for lighter η and K_L mesons. Yet, it is very important to take into account the form factor of the $\rho \to \pi\pi$ vertex for these mesons, because here the ρ-meson is far from its mass shell. In the $\omega \to 3\pi$ and $\phi \to 3\pi$ decays the form factor effect is less noticeable.

Finally, the authors express their gratitude to S. B. Gerasimov and A. B. Govorkov for useful discussions.

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Received on June 14, 1991.